An Attempt to Remove Quadratic Divergences in the Standard Theory

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The quadratic divergences caused by Yukawa interactions in the standard theory of elementary particle physics is shown to be removed by introducing finite-mass complex-ghost regulator fields. In this modification of the standard theory, its manifest covariance, renormalizability, gauge invariance and unitarity are retained, and no new observable particles are introduced.

1 Introduction

The standard theory of elementary particle physics (electroweak theory plus quantum chromodynamics) is a very successful theory. It is formulated as a *local* quantum field theory in 4-dimensional spacetime [1),2)] and its predictions have no clear contradictions with high-energy experimental results, provided that right-handed neutrinos are taken into account. This theory has the following fundamental properties:

1. Its Lagrangian density is manifestly covariant.
2. It is renormalizable.
3. It is invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge transformations.
4. Its physical S-matrix is unitary, though indefinite metric is used for BRS quantization of gauge fields.

The $SU(2)_L \times U(1)_Y$ gauge invariance is spontaneously broken up to $U(1)_{em}$ by Higgs mechanism. Higgs field has four real field-degrees of freedom; three of them are Nambu-Goldstone bosons, which are unphysical owing to the Kugo-Ojima subsidiary condition, and the remaining one is a massive scalar boson called Higgs boson. In order to give nonzero masses to leptons and quarks, it is assumed that there is a Yukawa interaction between Higgs field and each of them; the mass is essentially a product of the Yukawa coupling constant and the vacuum expecation value of Higgs field. This Yukawa interaction causes quadratic divergences in the self-energy part of Higgs boson.

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Because the standard theory is renormalizable, the renormalized S-matrix is finite to all orders of perturbation theory. But if one wishes to discuss radiative masses, one must introduce a cutoff parameter Λ. If Λ is taken to be of order of Planck mass, quadratically divergent quantities give uncontrollably large contributions. This trouble is known as the hierarchy problem. It may be resolved by the supersymmetric theory (SUSY), but since no superparticles are not yet observed at all, it is quite unlikely that SUSY is realized in Nature.

The purpose of the present paper is to propose a possible modification of the standard theory for removing the quadratic divergences caused by the Yukawa interactions, in such a way that the above-mentioned fundamental properies are retained and that no new observable particles are necessary to be introduced. As is well known, ultraviolet divergences can be removed by the regulator method (though it is quite nontrivial to remove all divergences so as to be consistent with non-abelian gauge invariance), but the introduction of indefinite metric usually violates the unitarity of the physical S-matrix. As was emphasized previously,[3),4)] however, complex-ghost quantum field theory does not violate the unitarity, because complex ghosts cannot appear [2] in the final state if they are absent in the initial state, owing to energy conservation law. Although Lorentz invariance of the S-matrix is known to be violated in this theory, the amount of the violation can be made small by appropriately choosing a certain parameter that characterizes the imaginary part of the complex-ghost mass. Accordinly, it is quite an attractive possibility to use complex ghosts as regulators. In the present paper, we show that the complex-ghost regulater method can remove the quadratic divergences caused by Yukawa interactions without violating the above-mentioned fundamental properties of the standard theory.

The present paper is organized as follows. In §2, the standard theory is very briefly reviewed. In §3, we present the theory of complex-ghost regulators. In §4, we show how the quadratic divergences caused by Yukawa interactions can be removed by the complex-ghost regulators.

2 Standard theory

The Lagrangian density of the standard theory consists of gauge-field part, lepton part, quark part and Higgs part. In the present paper, as we discuss Yukawa interactions, lepton part and quark part only are explicitly considered.

The lepton-part Lagrangian density is written as a sum over three generations $\alpha = e, \mu, \tau$.

[2]Precisely speaking, the probability with which a pair of a complex ghost and its complex-conjugate ghost appear is of measure *zero*.

In each generation, the left-handed charged lepton α_L and the left-handed neutrino $\nu_{\alpha L}$ constitute an $SU(2)_L$ doublet l_α with a hypercharge $Y = -1/2$, while the right-handed charged lepton α_R constitutes an $SU(2)_L$ singlet with $Y = -1$. Furthermore, recent neutrino oscillation experiments[5)] require the introduction of the right-handed neutrino $\nu'_{\alpha R}$, which is an $SU(2)_L$ singlet with $Y = 0$, and of the unitary neutrino-mixing matrix $U = (U_{\alpha\beta})$. We denote $\sum_\beta U_{\alpha\beta}\nu_\beta$ by $\tilde{\nu_\beta}$ and l_α with $\tilde{\nu_\alpha}$ in place of ν_α by $\tilde{l}_\alpha$; we set $\hat{l}_\alpha = (U^{-1}\tilde{l})_\alpha$. The Higgs field Φ is an $SU(2)_L$ doublet with $Y = 1/2$, and we set $\tilde{\Phi} = i\tau_2\Phi^*$.

The lepton-part Lagrangian density is given by

$$\begin{aligned}\mathcal{L}_{\text{lepton}} = \sum_{\alpha=e,\mu,\tau} \big[&\bar{\tilde{l}}_\alpha i\gamma^\mu \mathcal{D}_\mu^{L+Y}\tilde{l}_\alpha + \overline{r}_\alpha i\gamma^\mu \mathcal{D}_\mu^Y r_\alpha + \overline{r}'_\alpha i\gamma^\mu \partial_\mu r'_\alpha \\ &+(-f_\alpha)\{(\bar{\tilde{l}}_\alpha \Phi)r_\alpha + \overline{r}_\alpha(\Phi^\dagger \hat{l}_\alpha)\} + (-f'_\alpha)\{(\bar{\tilde{l}}_\alpha\tilde{\Phi})r'_\alpha + \overline{r}'_\alpha(\tilde{\Phi}^\dagger\tilde{l}_\alpha)\}\big],\end{aligned} \tag{2.1}$$

where $\mathcal{D}_\mu^{L+Y}$ and $\mathcal{D}_\mu^Y$ denote covariant differentiations with respect to $SU(2)_L \times U(1)_Y$ and $U(1)_Y$, respectively.

The quark-part Lagrangian density has essentially the same form as the lepton-part one does, except for the fact that the former has the color degrees of freedom, which is inessential to the present work. The "upper" quarks $\{u, c, t\}$ and the "lower" quarks $\{d, s, b\}$ correspond to the charged antileptons $\{\overline{e}, \overline{\mu}, \overline{\tau}\}$ and to the antineutrinos $\{\overline{\nu}_e, \overline{\nu}_\mu, \overline{\nu}_\tau\}$, respectively. In this correspondence, the values of the hypercharge should be changed as $Y_{\text{quark}} = Y_{\text{antilepton}} - 1/3$.

The lower component of the Higgs field Φ acquires a nonvanishing vacuum expectation value $v/\sqrt{2}$ $(v > 0)$; hence the vacuum expectation value of the upper component of $\tilde{\Phi}$ is also $v/\sqrt{2}$. If Φ is reexpressed in terms of v and four hermitian fields (Higgs boson and 3-component Nambu-Goldstone boson), the quadratic part of $\mathcal{L}_{\text{lepton}}$ becomes the free Dirac Lagrangian density for all leptons, modified by neutrino mixing. The mass m_α of a charged lepton α is given by $f_\alpha v/\sqrt{2}$. As for neutrinos, we encounter neutrino-mixing mass matrix, $\mathcal{M}$. Likewise for $\mathcal{L}_{\text{quark}}$.

3 Complex-ghost regulators

We introduce pairs of Weyl-spinor fields L_j ($SU(2)_L$ doublet, $Y = -1/2$), R_j ($SU(2)_L$ singlet, $Y = -1$) and R'_j ($SU(2)_L$ singlet, $Y = 0$) $(j = 1, 2)$; the $j = 1$ fields have positive norm, while the $j = 2$ ones have negative norm. Imitating (2.1), we introduce the Lagrangian

density

$$\begin{aligned}\mathcal{L}_{\text{regulator}} = &\sum_{j=1,2}(-1)^{j-1}\left[\overline{L}_j i\gamma^\mu \mathcal{D}_\mu^{L+Y} L_j + \overline{R}_j i\gamma^\mu \mathcal{D}_\mu^{Y} R_j + \overline{R}'_j i\gamma^\mu \partial_\mu R'_j\right] \\ &+ \sum_{j,k=1}^{2}\left[(-f_{jk})\{(\overline{L}_j \Phi)R_k + \overline{R}_j(\Phi^\dagger L_k)\} + (-f'_{jk})\{(\overline{L}_j\tilde{\Phi})R'_k + \overline{R}'_j(\tilde{\Phi}^\dagger L_k)\}\right],\end{aligned} \tag{3.1}$$

where $f_{jk} = f^*_{kj}$ and $f'_{jk} = {f'_{kj}}^*$.

Φ is reexpressed in terms of v and four real fields. We set

$$\begin{aligned} &f_{11}v/\sqrt{2} = m_1, \quad f_{22}v/\sqrt{2} = -m_2, \quad f_{12}v/\sqrt{2} = \gamma/2; \\ &f'_{11}v/\sqrt{2} = m'_1, \quad f'_{22}v/\sqrt{2} = -m'_2, \quad f'_{12}v/\sqrt{2} = \gamma'/2. \end{aligned} \tag{3.2}$$

Then the quadratic part of (3.1) becomes

$$\begin{aligned}\mathcal{L}^0_{\text{regulator}} = &\sum_{j=1}^{2}(-1)^{j-1}(\overline{\Psi}_j i\gamma^\mu\partial_\mu\Psi_j - m_j\overline{\Psi}_j\Psi_j) - \frac{\gamma}{2}\overline{\Psi}_1\Psi_2 - \frac{\gamma^*}{2}\overline{\Psi}_2\Psi_1 \\ &+ \sum_{j=1}^{2}(-1)^{j-1}(\overline{\Psi}'_j i\gamma^\mu\partial_\mu\Psi'_j - m'_j\overline{\Psi}'_j\Psi'_j) - \frac{\gamma'}{2}\overline{\Psi}'_1\Psi'_2 - \frac{{\gamma'}^*}{2}\overline{\Psi}'_2\Psi'_1,\end{aligned} \tag{3.3}$$

where Ψ_j is composed of the upper component of L_j and R_j and Ψ'_j is composed of the lower component of L_j and R'_j.

The field equations derived from (3.3) are

$$\begin{aligned}(i\gamma^\mu\partial_\mu - m_1)\Psi_1 - \frac{\gamma}{2}\Psi_2 = 0, \\ (i\gamma^\mu\partial_\mu - m_2)\Psi_2 + \frac{\gamma^*}{2}\Psi_1 = 0.\end{aligned} \tag{3.4}$$

Canonical quantization is performed with *"wrong statistics"*, that is, we set up *commutation relations* but not anticommutation relations:

$$\begin{aligned}[\Psi_j(x), \overline{\Psi}_k(y)]_{x_0=y_0} &= (-1)^{j-1}\gamma^0\delta(\boldsymbol{x}-\boldsymbol{y}) \quad (j=k) \\ &= 0 \quad (j \neq k).\end{aligned} \tag{3.5}$$

Then we set up a Cauchy problem for the 4-dimensional commutators

$$[\Psi_j(x), \overline{\Psi}_k(y)] \equiv iS_{jk}(x-y). \tag{3.6}$$

As was done previously,[6),7)] the solution to the Cauchy problem is easily found by diagonalizing the mass matrix

$$\boldsymbol{M} = \begin{pmatrix} m_1 & \gamma/2 \\ -\gamma^*/2 & m_2 \end{pmatrix}. \tag{3.7}$$

The eigenvalues become complex if

$$|\gamma| > |m_1 - m_2|. \tag{3.8}$$

Then we obtain the complex masses M and M^*, where

$$M \equiv \frac{m_1+m_2}{2} + \frac{i}{2}\sqrt{\gamma\gamma^* - (m_1-m_2)^2}. \tag{3.9}$$

Hence,

$$M^2 + M^{*2} = m_1^2 + m_2^2 - \frac{\gamma\gamma^*}{2} = \mathrm{tr}\boldsymbol{M^2}. \tag{3.10}$$

We can then construct the Wightman functions $S_{jk}^{(+)}(x-y)$ and the Feynman propagator $S_{Fjk}(x-y)$ quite analogously to the case of the scalar complex ghost. But we do not need their explicit expressions for the present purpose.

The same procedure as above is carried out for primed quantities. That is, corresponding to (3.4) - (3.10), the equations in which Ψ_j, m_j, γ, S_{jk}, $\boldsymbol{M}$ and M are replaced by the respective primed ones hold.

Furthermore, for the quark part, everything goes quite analogously to the above consideration on the lepton part. Of course, in the quark case, the complex-ghost regulators have the color degrees of freedom.

4 Removal of quadratic divergences

As is well known, the appearance of quadratic divergences in the standard theory is only in the proper self-energy part of Higgs boson. In the present paper, we discuss the quadratic divergences caused by the Yukawa interaction. That is, we consider only the self-energy Feynman diagrams in which the end vertices of both external Higgs-boson lines correspond to Yukawa interactions.

The internal-line part of such a Feynman diagram consist of a lepton or quark loop together with radiative corrections. Because of the Fermi statistics of leptons and quarks, this loop gives an overall factor -1 to the Feynman integral. On the other hand, *the Feynman diagram that has a complex-ghost regulator loop acquires no such a factor because of the Bose statistics of the regulator fields.* Furthermore, it is important to note that as far as quadratic divergences are concerned, *the mass term of any Feynman propagator is irrelevant*; that is, we may forget about the mass except for the fact that each Yukawa coupling constant is proportional to a mass (or mass matrix element).

We first consider the second-order self-energy parts. The contribution from the charged lepton-loop diagrams is proportional to $-\sum_\alpha m_\alpha^2$. The contribution from the corresponding regulator-loop diagrams is proportional to

$$f_{11}^2 + 2(-1)f_{12}f_{21} + (-1)^2 f_{22}^2 = \left(\frac{v}{\sqrt{2}}\right)^{-2}\left(m_1^2 + m_2^2 - \frac{\gamma\gamma^*}{2}\right). \tag{4.1}$$

Here the factor -1 is due to the indefinite metric appearing in (3.3). Hence the quadratic divergence caused by three charged leptons is cancelled if

$$\sum_{\alpha} m_\alpha^2 = M^2 + M^{*2} \tag{4.2}$$

Likewise, the quadratic divergence caused by three neutrinos is cancelled if

$$\mathrm{tr}\mathcal{M}^2 = M'^2 + M'^{*2}, \tag{4.3}$$

where $\mathcal{M}$ denotes the neutrino-mixing mass matrix.

Unfortunately, for higher-order self-energy diagrams, such a simple cancellation condition as above is no longer valid. In order to realize the removal of quadratic divergences to all orders, we must introduce complex-ghost regulator fields for *each* generation. That is, we must replace (3.1) by

$$\begin{aligned}\mathcal{L}_{\mathrm{regulator}} = \sum_{\alpha=e,\mu,\tau} \Big(& \sum_{j=1,2} (-1)^{j-1} \big[\overline{\tilde{L}}_{\alpha j} i\gamma^\mu \mathcal{D}_\mu^{L+Y} \tilde{L}_{\alpha j} + \overline{R}_{\alpha j} i\gamma^\mu \mathcal{D}_\mu^{Y} R_{\alpha j} + \overline{R}'_{\alpha j} i\gamma^\mu \partial_\mu R'_{\alpha j}\big] \\ & + \sum_{j,k=1}^{2} \big[(-f_{\alpha jk})\{(\overline{\hat{L}}_{\alpha j}\Phi)R_{\alpha k} + \overline{R}_{\alpha j}(\Phi^\dagger \hat{L}_{\alpha k})\} \\ & \quad + (-f'_{\alpha jk})\{(\overline{\tilde{L}}_{\alpha j}\tilde{\Phi})R'_{\alpha k} + \overline{R}'_{\alpha j}(\tilde{\Phi}^\dagger \tilde{L}_{\alpha k})\}\big]\Big),\end{aligned} \tag{4.4}$$

where notation will be obvious from the above consideration.

Then, for example, for a charged lepton α, (4.2) should be replaced by

$$m_\alpha^2 = M_\alpha^2 + M_\alpha^{*2} \quad \text{for each } \alpha, \tag{4.5}$$

where M_α denotes the complex mass acquired by $\Psi_{\alpha j}$. As for the neutrino-loop self-energy part, a similar relation should be set up for each eigenvalue of the mass matrix.

For the quark part, everything is analogous to the above.

Thus, the quadratic divergences caused by Yukawa interactions can be removed by the introduction of the complex-ghost regulator fields without violating the unitarity of the physical S-matrix, though Lorentz invariance is spontaneously violated slightly.

References

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